Single View 3D Reconstruction under an Uncalibrated Camera and an Unknown Mirror Sphere

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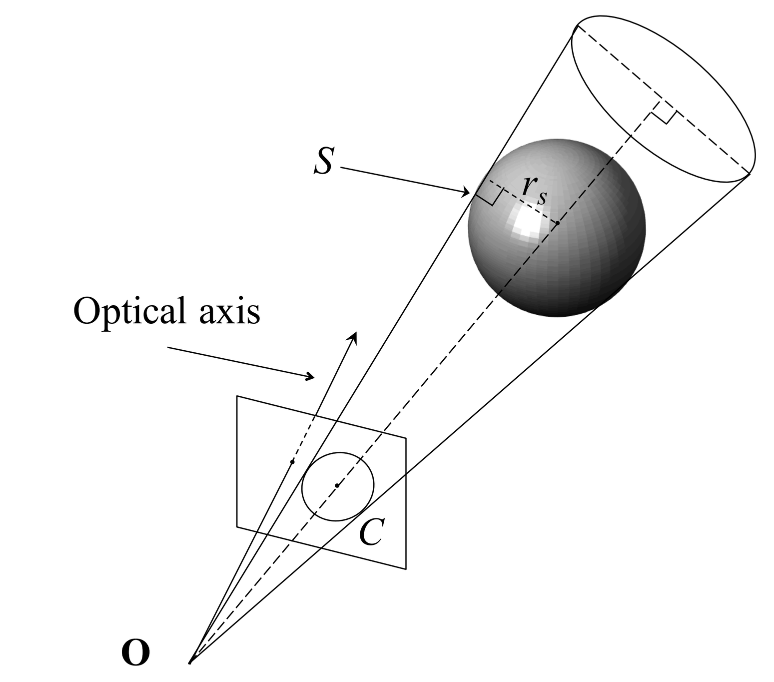
Problem Definition

We address the problem of single view 3D reconstruction using a mirror sphere, and propose a novel self-calibration method. Unlike other mirror sphere based reconstruction methods, our method needs neither the intrinsic parameters of the camera, nor the position and radius of the sphere be known.

The key contributions of this work are:

- The first single view 3D reconstruction method that works under an uncalibrated camera and an unknown mirror sphere.
- An analytical solution for recovering the focal length of a camera from an image of an unknown sphere given the principal point of the camera.
- A robust method for estimating both the principal point and focal length of a camera from multiple images of an unknown sphere placed at different positions.
- A novel method for estimating both the principal point and focal length of a camera from just one single image of an unknown sphere.

Estimating Camera Intrinsic Parameters



The image of a sphere forms a conic, which can be represented by a 3x3 symmetric matrix **C**.

Theoretical Background

By removing the effect of **K**, conic **C** will transform into $\hat{\mathbf{C}} = \mathbf{K}^{\mathrm{T}}\mathbf{C}\mathbf{K}$.

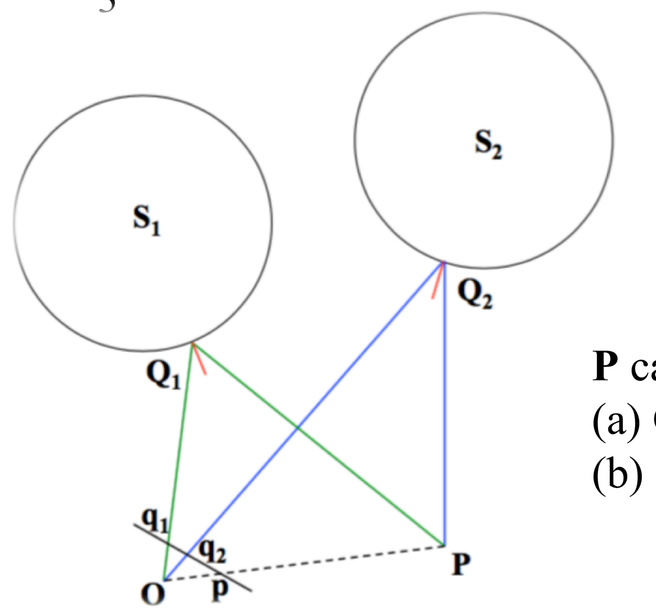
By eigen decomposition, $\hat{\mathbf{C}} = \mathbf{R}\mathbf{D}\mathbf{R}^{\mathrm{T}}$, where **R** is the camera rotation matrix, and $\mathbf{D} = diag(\lambda_1, \lambda_1, \lambda_2)$.

D represents a circle of radius $r_c = \sqrt{-\frac{\lambda_1}{\lambda_2}}$ centres at the image plane origin.

The sphere centre can be recovered as $d\mathbf{r}_3$, where $\mathbf{r}_3$ is the third column of **R** and $d = r_s\sqrt{\frac{1+r_c^2}{r_c}}$.

Shape Recovery

With a calibrated camera and a sphere with known radius, the position of the sphere can be uniquely recovered from its image. An object can be reconstructed from its reflections on the sphere placed at two distinct positions.



P can be reconstructed by triangulating:
(a) $\mathbf{Q}_1\mathbf{P}$ with $\mathbf{Q}_2\mathbf{P}$, if **P** is not visible;
(b) **OP** with $\mathbf{Q}_1\mathbf{P}$ (or $\mathbf{Q}_2\mathbf{P}$), if **P** is visible.

Analytical solution for *f*

Let $\mathbf{K} = \mathbf{T}\mathbf{F}$, where $\mathbf{T} = \begin{bmatrix} 1 & 0 & u_0 \\ 0 & 1 & v_0 \\ 0 & 0 & 1 \end{bmatrix}$, and $\mathbf{F} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$

=> $\hat{\mathbf{C}} = \mathbf{F}^{\mathrm{T}}\bar{\mathbf{C}}\mathbf{F}$, where $\bar{\mathbf{C}} = \mathbf{T}^{\mathrm{T}}\mathbf{C}\mathbf{T}$

=> $det(\hat{\mathbf{C}} - \lambda\mathbf{I}) = 0$

=> $\lambda^3 - \beta\lambda^2 + \gamma\lambda + \delta = 0$

where λ is the eigen value of $\hat{\mathbf{C}}$, β, γ, δ are non-linear combination of terms in $\hat{\mathbf{C}}$

=> $\mu^2 - 4\nu^3 = 0$

where $\mu = \beta^2 - 3\gamma$, $\nu = 2\beta^3 - 9\beta\gamma - 27\delta$

Given **C** and principal point, the only unknown is *f*. Then *f* can be obtained by solving the equation.

Robust method for *f* and (u_0, v_0)

Algorithm 1: Estimation of the principal point and focal length from two conic images of a mirror sphere.

Input : Image centre (u_c, v_c), conic images $\mathbf{C}_1, \mathbf{C}_2$
Output: Principal point (u_0, v_0), focal length f
Initialization: Set offsets along u, v directions as $w = const1$, $h = const2$; set step size as $s = const3$; set error as $error_{min} = large\ const$;
for $u_p \leftarrow u_c - w$ **to** $u_c + w$ **step** s **do**
  for $v_p \leftarrow v_c - h$ **to** $v_c + h$ **step** s **do**
    Construct **T** with (u_p, v_p);
    Compute $\bar{\mathbf{C}}_1, \bar{\mathbf{C}}_2$;
    Construct $\hat{\mathbf{C}}_1, \hat{\mathbf{C}}_2$ from $\bar{\mathbf{C}}_1, \bar{\mathbf{C}}_2$;
    Estimate f_1, f_2 with $\hat{\mathbf{C}}_1, \hat{\mathbf{C}}_2$;
    Compute current error, $error_{cur}$;
    if $error_{cur} < error_{min}$ **then**
      $u_0 \leftarrow u_p$
      $v_0 \leftarrow v_p$
      $f \leftarrow \frac{|real(f_1)| + |real(f_2)|}{2}$
      $error_{min} \leftarrow error_{cur}$
    end
  end
end

Single image solution

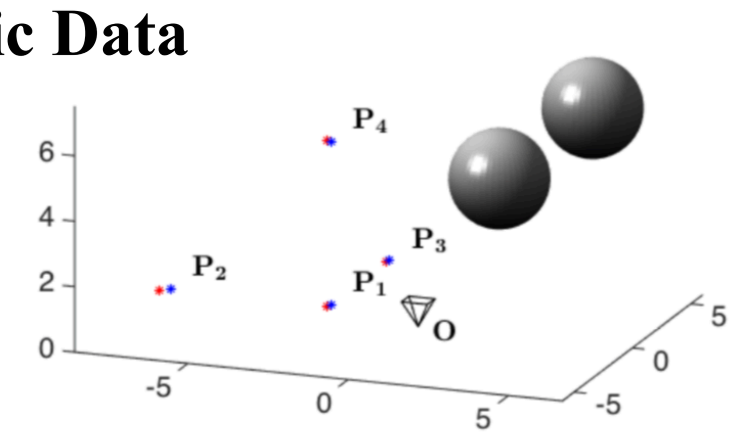
Step1: sample points evenly within a small window centred at the image centre, and estimate a focal length *f* using each sample point as the principal point.

Step2: calculate the mean of these estimated values and this gives us a final estimate of the focal length.

Step 3: identify the sample point that leads to an estimated focal length closest to the mean value as the principal point.

Experimental Results

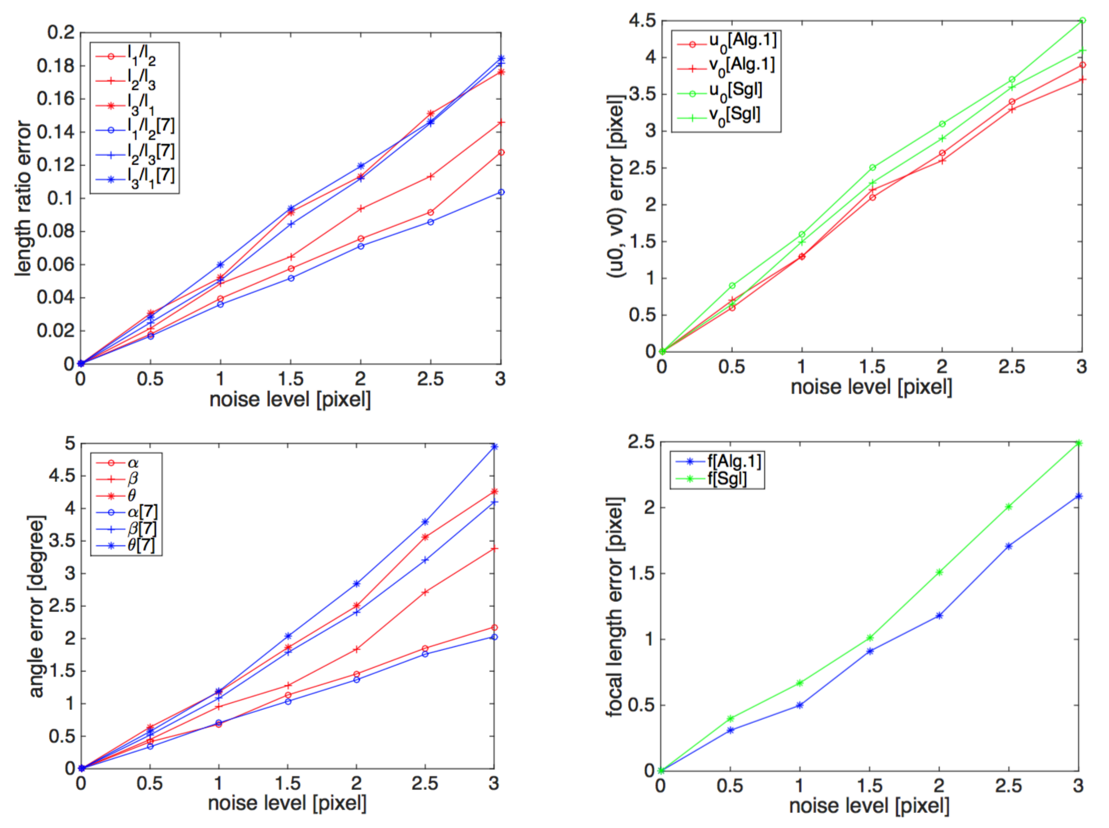
Synthetic Data



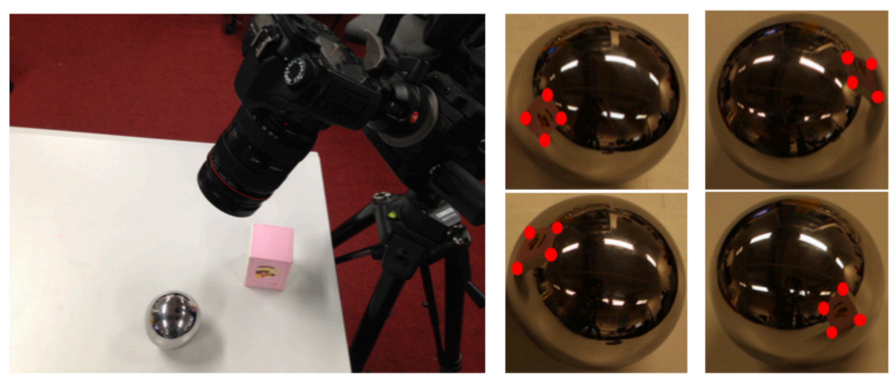
Blue: ground truth
Red: ours

$l_1 = \|\mathbf{P}_1\mathbf{P}_2\|$
$l_2 = \|\mathbf{P}_1\mathbf{P}_3\|$
$l_3 = \|\mathbf{P}_1\mathbf{P}_4\|$

$\alpha = \angle\mathbf{P}_2\mathbf{P}_1\mathbf{P}_3$
$\beta = \angle\mathbf{P}_3\mathbf{P}_1\mathbf{P}_4$
$\theta = \angle\mathbf{P}_4\mathbf{P}_1\mathbf{P}_2$



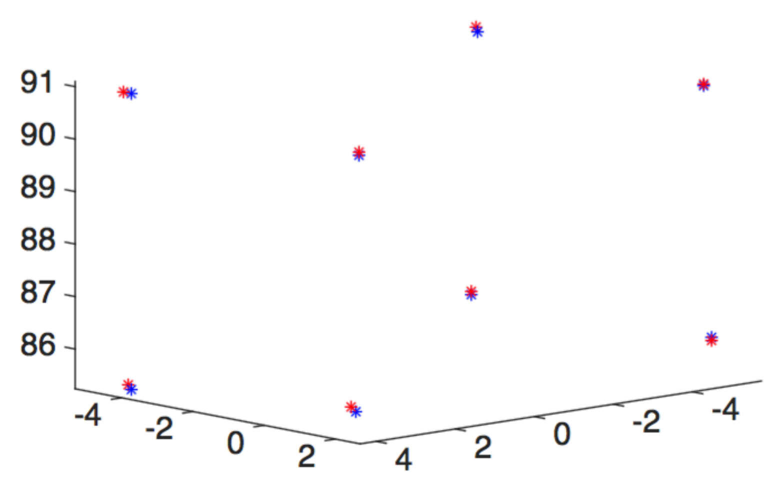
Real Data



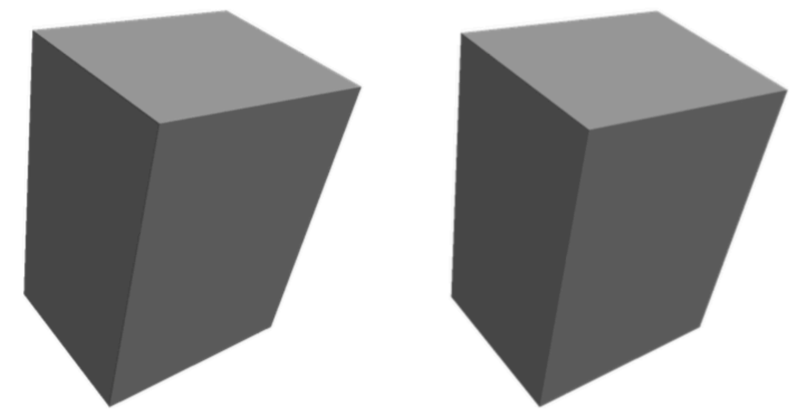
Experimental setup

	f	u_0	v_0
Calibration [5]	4435.36	1963.0	1277.0
Estimation [Centre]	4117.49	1944.0	1296.0
Estimation [Alg.1]	4386.06	1955.0	1285.0
Estimation [Sgl]	4301.02	1980.0	1290.0
Error [Centre]	7.17%	0.97%	1.49%
Error [Alg.1]	**1.11%**	**0.41%**	**0.63%**
Error [Sgl]	3.06%	0.87%	1.02%

	angle	*length ratio*
Calibrated[7]	1.08	0.03
Ours	1.05	0.03



Blue: calibrated[7]; Red: ours



Calibrated[7] Ours

[5] J.-Y. Bouguet. Camera calibration toolbox for matlab. http://www.vision.caltech.edu/bouguetj/calib doc/.
[7] Z. Chen, K.-Y. K. Wong, M. Liu, and D. Schnieders. Single- view reconstruction from an unknown spherical mirror. In *ICIP*, pages 2733–2736, 2011

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